

NONPERTURBATIVE METHODS IN THE PROBLEM OF MULTIPHOTON EXCITATION OF ATOM BY SQUEEZED LIGHT

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Abstract

The multiphoton detectors for the strong squeezed light vacuum are considered. The result is compared with the perturbation theory. It is shown that as the degree of squeezing is increased the statistical factor decreases.

Multiphoton transitions in atoms due to squeezed light were analyzed for the first time by Yansky and Yushin [1] by using perturbation theory. On the other hand, at present parametric generators of squeezed light are discussed [2]. They allow us to obtain high density of photons $N \sim 10^{20} - 10^{21}$ in resonator with volume $V \sim 1 \text{ cm}^3$ for stored energy density $\geq 1 \text{ J}$. Although experimentally such photon densities are not reached, it is of interest to describe physical processes in atoms interesting with intensive squeezed light. For the squeezed vacuum $|0\rangle_s$, as is known, $N = \langle 0|a^+a|0\rangle_s = |\nu|^2(a^+(a) - \text{are operators of appearing and disappearing of quantum of electromagnetic field}), \nu = |\nu|e^{i\phi}$ is squeezing parameter of Stoler unitary transformation [3, 4] of operators $a^+(a)$ to the new variables of squeezed field $b^+(b)$:

$$\begin{aligned} b &= \mu a + \nu a^+ \\ b^+ &= \mu^* a^+ + \nu^* a; |\mu|^2 - |\nu|^2 = 1 \end{aligned} \quad (1)$$

For the squeezing degree $\nu \sim 10^{10} - 10^{11}$ the criteria for application of perturbation theory methods are not satisfied. In fact, let us consider two level system with nonzero average dipole moment d in the excited state (2) (neglect for simplicity the dipole moment in the ground state (1)). The characteristic theory parameter ρ appearing due to multiphoton transition on the degenerate level (2) has the form [5]

$$\rho = Fd/\hbar\omega \quad (2)$$

where F is the amplitude of the intensity of electromagnetic field with frequency ω . Parameter $\rho \geq q_0$ (q_0 is the number of photons participating in the transition) is reached for $N \sim 10^{20} - 10^{21}$ ($q_0 \sim 3 - 5, d \sim 10D$).

In the paper [6] the statistical factor $\chi_{(G\delta)} = W^{(G)}/W^{(\delta)}$ was calculated for the multiphoton transition on the degenerated level of hydrogen atom for the source of gauss electromagnetic field (G) and pure coherent source (δ). It was shown that with the increase of radiation intensity the difference in statistical properties of multiphoton excitation of atom disappear. The expression was

received for the probability of coherent multiphoton transition in the presence of probe radiation with intensity \mathcal{F} and frequency $\Omega \gg \omega$:

$$W^{(\delta)}(\Omega) = \frac{d_{12}^2 \mathcal{F}^2}{\hbar^2 \gamma} J_n^2(\rho) \quad (3)$$

where $q_0 = (\Delta - \hbar\Omega)/\hbar\omega$, Δ is electron excitation energy, γ is damping constant of excited electron state, $J_n(x)$ is the Bessel function of real argument. Using the formula (3) gives us methodical advantage because it permits to realize the rearrangement of multiphoton process with the frequency of probe radiation. Let us consider the statistical factor $\chi_{(S\delta)} = W^{(S)}/W^{(\delta)}$, where W^S is the transition probability under the action of squeezed light. S - matrix formalism is used for calculating W^S . Confining to the second order of perturbation theory on the probe radiation. We have:

$$W^S(\Omega) = \frac{d_{12}^2 \mathcal{F}^2}{\hbar^2} \int_{-\infty}^{+\infty} dt \exp[iq_0 \omega t - \gamma t] I^s(t) \quad (4)$$

where $I^s(t)$ is generating function of transition probability:

$$I^s(t) = \langle G(t) \rangle_s$$

The evolution operator $G(t)$ satisfied the motion equation:

$$i\hbar \dot{G}(t) = [g(t)a + g^*(t)a^+]G(t); G(0) = 1 \quad (5)$$

$$g(t) = i v e^{-i\omega t}; v = d_{22}(2\pi\hbar\omega/V)^{1/2}$$

The brackets $\langle \dots \rangle$ in (5) denote the averaging over squeezed state, d_{22} in (5) is dipole moment in electronic state (2), $d_{22} \approx 10e_0 a_0$ for the level with the main quantum number $n = 3$ (a_0 is the Bohr radius, e_0 is the electron charge). The solution to (5) may be presented in the following normally ordered form [7]:

$$\begin{aligned} G(t) &= e^{A(t)} e^{-B^*(t)a^+} e^{B(t)a} \\ B(t) &= -\frac{i}{\hbar} \int_0^t d\tau g(\tau) \\ A(t) &= -\frac{1}{\hbar^2} \int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2 g(\tau_1) g^*(\tau_2) \end{aligned} \quad (6)$$

Let us use back transition to (1):

$$\begin{aligned} a &= \mu^* b - \nu b^+ \\ a^+ &= \mu b^+ - \nu^* b \end{aligned}$$

With the Backer - Hausdorff transformation

$$\begin{aligned} e^A \cdot e^B &= e^{A+B} \cdot e^{\frac{1}{2}[A,B]} \\ [[A, B], A] &= [[A, B], B] = 0 \end{aligned}$$

it is easy to receive the following expression for the generating function $I^{(s)}$ of quantum transition under the action of squeezed light

$$\begin{aligned} I^{(s)}(t) &= I_0^{(s)}(t) \cdot \langle \beta | e^{-(B\nu + B^*\mu)b^+} \cdot e^{(B^*\nu^* + B\mu^*)b} | \beta \rangle, \\ I_0^{(s)}(t) &= \exp \left\{ -|\nu|^2 |B|^2 - \frac{1}{2} (B^2 \mu \nu^* + \text{c.c.}) \right\} \end{aligned} \quad (7)$$

The value β characterizes the initial coherent state $|\beta\rangle$. In the case of squeezed vacuum we have $I^{(s)}(t) = I_0^{(s)}(t)$. The received exact expression for the generating function $I^{(s)}(t)$ does not permit to make analytical calculation of the transition probability and creates certain difficulties for numerical calculations. This expression differs from the known formulas in [7,8] obtained in perturbation theory in two positions. Firstly, in (7) the reemitting of photons is taken into account, secondly, anomaly correlation functions with nonequal number of operators a and a^+ are not discarded. The first condition for the strong field is strictly necessary. The second condition may be used for both weak and strong fields, as will be shown below. Taking into consideration the remarks let us simplify the common expression for the transition probability. Present formula (6) in antinormal form and rewrite $I_0^{(s)}(t)$:

$$\begin{aligned} I_0^{(s)}(t) &= e^{-|B|^2} \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} |B|^{2m} \cdot \langle 0 | a^m a^{+m} | 0 \rangle + \\ &+ e^{-|B|^2} \sum_{m \neq n} \frac{(-1)^m}{m!n!} B^{*m} B^n \cdot \langle 0 | a^n a^{+m} | 0 \rangle, \end{aligned} \quad (8)$$

The presentation of the evolution operator $G(t)$ in antinormal form is caused by simplicity of calculations, for example:

$$\langle 0 | a a^+ | 0 \rangle = 1 + |\nu|^2 = |\mu|^2 = N + 1$$

The last term in (8) is the contribution of anomaly correlation functions and do not gives the contribution in multi-photon processes. Thus, we leave the first member in (8). We find:

$$I_0^{(s)}(t) = e^{-|\nu|^2 |B|^2} \cdot I_0(|B|^2 \mu |\nu|) \quad (9)$$

where $I_0(x)$ is modified Bessel function. Let us consider the photon density $|\nu| \gg 1$ corresponding to perturbation theory. In this case (see Appendix) it may be shown that statistical factor ($\chi_{(s\delta)}$)

$$\chi_{(s\delta)} = \frac{W^{(s)}}{W^{(\delta)}} = (2q_0 - 1)!! \quad (10)$$

This result coincides with the known conclusion in [1]. In Fig.1 the calculation of statistical factor $\chi_{(s\delta)}$ in nonperturbative approach is given. Dashed line corresponds to perturbation theory. For comparison the same Fig.1 gives the statistical factor $\chi_{(Q\delta)}$. Naturally, near field intensity which corresponds to the suppression coherent multiphoton excitation effect [10], the statistical factor increases drastically, which creates additional possibilities for experiment.

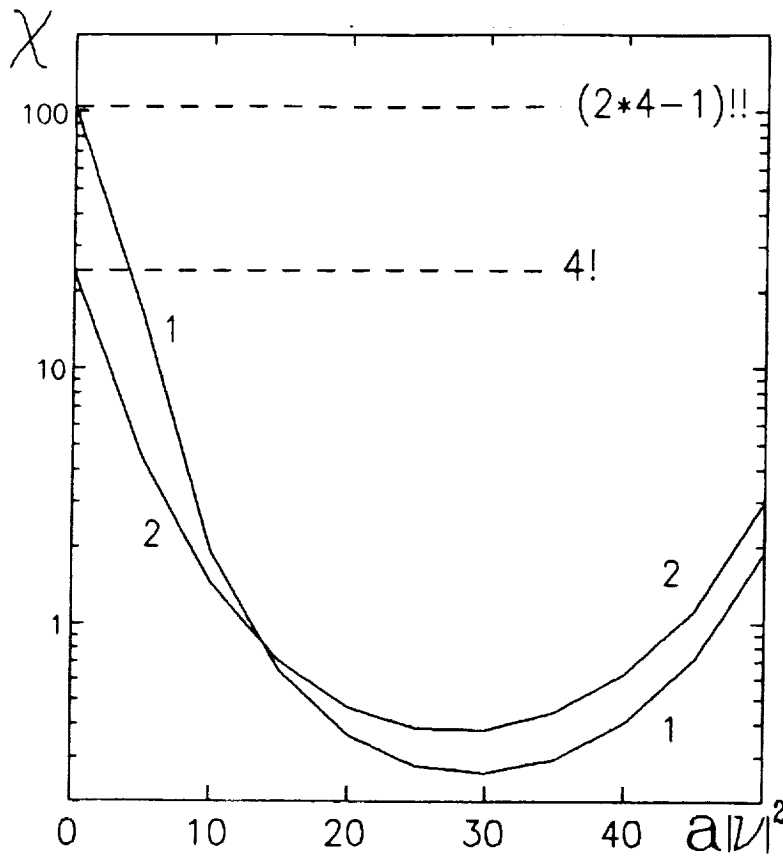


Fig.1. Dependence of χ on $a|\nu|^2$. Curve 1 corresponds $\chi_{(s)}$; 2 - $\chi_{(s)}$

Appendix

Let us use the expression for the multiplication of coherent state $|\alpha\rangle$ and squeezed state $|\beta\rangle$,

$$\langle \alpha | \beta \rangle = \frac{1}{\sqrt{\mu}} \exp \left\{ -\frac{1}{2} (|\alpha|^2 + |\beta|^2 + \frac{\nu}{\mu} \alpha^{*2} + \frac{\nu^*}{\mu} \beta^2) + \frac{1}{\mu} \alpha^* \beta \right\}$$

Rewrite A_{mm} as:

$$\begin{aligned} A_{mm} &= \frac{1}{\pi} \int d^2\alpha |\alpha|^{2m} \langle 0 | \alpha \rangle^2 = \\ &= \frac{1}{\mu} \int_0^\infty dx e^{-x} x^m I_0(x \frac{|\nu|}{\mu}) \end{aligned}$$

$(d^2\alpha = d(\text{Re}\alpha)d(\text{Im}\alpha))$ - is the measure of integration in complex plane α . So, the generating function may have the form:

$$I^{(s)}(t) = \frac{1}{\mu} \int_0^\infty dx e^{-x} I_0\left(x \frac{|\nu|}{\mu}\right) J_0(2|B|\sqrt{x})$$

After calculating this integral we obtain the generating function (9). Let us use the summation formula for the Bessel function [11]:

$$J_0(2a \sin x/2) = \sum_{k=-\infty}^{+\infty} J_k^2(a) e^{ikx}$$

Let write the expression for the multiphoton transition probability $W^{(s)}$ under the action of the squeezed light:

$$W^{(s)} \sim \frac{1}{\mu} \sum_m F_m \delta(q_0 - m) \simeq \frac{1}{\mu} F_{q_0},$$

here we denote:

$$F_{q_0} = \int_0^\infty dx e^{-x} I_0\left(x \frac{|\nu|}{\mu}\right) J_{q_0}^2(2\sqrt{ax})$$

$$a = v^2/(\hbar\omega)^2$$

The last integral is known [11]. We receive:

$$W^{(s)} \sim \frac{1}{\mu} \frac{a^{q_0}}{(q_0!)^2} \sum_{k=0}^{\infty} \frac{1}{2^{2k}(k!)^2} \left(\frac{|\nu|}{\mu}\right)^{2k} (2k + q_0)! \times$$

$$\times {}_2F_2(q_0 + 1/2, q_0 + 2k + 1; q_0 + 1, 2q_0 + 1; -4a),$$

where ${}_2F_2$ is the common hipergeometrical series. At $a \ll 1$, ${}_2F_2 \sim 1$. We use the integral representation for the factorial. It is possible to sum up the series:

$$W^{(s)} \sim \frac{(a\mu)^{q_0}}{q_0!} \cdot P_{q_0}(\mu)$$

$P_{q_0}(\mu)$ is the Legendre polynomial. In the approximation $|\nu| \gg 1$ ($\mu \gg 1$). Let us use the asymptotic expression [11]:

$$P_{q_0}(\mu) \approx \frac{(2q_0 - 1)!!}{q_0!} \mu^{q_0}$$

We receive:

$$W^{(s)} \sim \frac{(a\mu^2)^{q_0}}{(q_0!)^2} (2q_0 - 1)!! = W^{(s)}(2q_0 - 1)!!$$

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